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Non-Fermi Liquid Effective Field Theory of Dense QCD Matter

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We review an effective field theory for the non-Fermi liquid regime of dense QCD matter. Non-Fermi liquid effects arise due the presence of unscreened magnetic gluon exchanges. We show that there is a systematic low energy expansion in fractional powers and logarithms of energy. We discuss the validity of some standard theorems of Fermi liquid theory.

1. Non-Fermi liquid effective field theory

At high baryon density the relevant degrees of freedom are particle and hole excitations which move with the Fermi velocity v. Since the momentum $p \sim v\mu$ is large, typical soft scatterings cannot change the momentum by very much and the velocity is approximately conserved. An effective field theory of particles and holes in QCD is given by [1, 2]

$$\mathcal{L} = \psi_v^\dagger \left(iv \cdot D - \frac{1}{2p_F} D_\perp^2 \right) \psi_v + \mathcal{L}_{4f} + \mathcal{L}_{HDL} - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \ldots, \tag{1}$$

where $v_\mu = (1, \vec{v})$. The field ψ_v describes particles and holes with momenta $p = \mu(0, \vec{v}) + k$, where $k \ll \mu$. We will write $k = k_0 + k_\parallel + k_\perp$ with $\vec{k}_\parallel = \vec{v}(\vec{k} \cdot \vec{v})$ and $\vec{k}_\perp = \vec{k} - \vec{k}_\parallel$. $\mathcal{L}_{4f}$ denotes four-fermion operators in the BCS and zero sound channel. At energies below the screening scale $g\mu$ hard dense loops have to be resummed. The generating functional for hard dense loops in gluon n-point functions is given by [3]

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \int \frac{d\hat{v}}{4\pi} G_{\mu\alpha}^a \frac{v^\alpha v^\beta}{(v \cdot D)^2} G_{\mu\beta}^b, \tag{2}$$

where $m^2 = N_f g^2 \mu^2 / (4\pi^2)$ is the dynamical gluon mass and $\hat{v}$ is a unit vector in the direction of $\vec{v}$.

The hard dense loop action describes static screening of electric fields and dynamic screening of magnetic modes. Since there is no screening of static magnetic fields low energy gluon exchanges are dominated by magnetic modes. The resummed transverse gauge boson propagator is given by

$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\eta k_0 / |\vec{k}|}, \tag{3}$$

where $\eta = \frac{\pi}{2} m^2$ and we have assumed that $|k_0| < |\vec{k}|$. We observe that the gluon propagator becomes large in the regime $|\vec{k}| \sim (\eta k_0)^{1/3} \gg k_0$. This leads to an unusual scaling

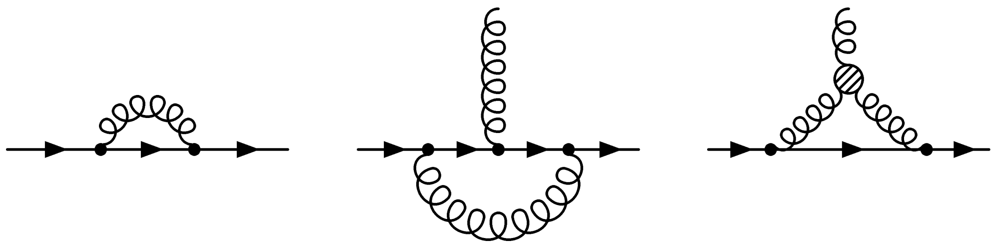

Figure 1. One-loop contributions to the quark self energy and the quark-gluon vertex. In the magnetic regime the graphs scale as $\omega\log(\omega)$, $\omega^{1/3}$ and $\omega^{2/3}$, respectively.

behavior of Green functions in the low energy limit. Consider a generic Feynman diagram and scale all energies by a factor s. Because of the functional form of the gluon propagator in the Landau damped regime gluon momenta scale as $|\vec{k}|\sim s^{1/3}$. This implies that the gluon momenta are much larger than the gluon energies. The quark dispersion relation is $k_0\simeq k_{||}+k_\perp^2/(2p_F)$. The only way a quark can emit a very spacelike gluon and remain close to the Fermi surface is if the gluon momentum is transverse to the Fermi velocity. We find

$$k_0\sim s, \quad k_{||}\sim s^{2/3}, \quad k_\perp\sim s^{1/3}, \tag{4}$$

and $k_0\ll k_{||}\ll k_\perp$. In the low energy regime propagators and vertices can be simplified even further. The quark and gluon propagators are

$$S^{\alpha\beta}(p)=\frac{i\delta_{\alpha\beta}}{p_0-p_{||}-\frac{p_\perp^2}{2\mu}+i\epsilon\,\mathrm{sgn}(p_0)}, \qquad D_{ij}(k)=\frac{-i\delta_{ij}}{k_\perp^2-i\frac{\pi}{2}m^2\frac{k_0}{k_\perp}}\,, \tag{5}$$

and the quark gluon vertex is $gv_i(\lambda^a/2)$. Higher order corrections can be found by expanding the quark and gluon propagators as well as the HDL vertices in powers of the small parameter $\epsilon\equiv\omega/m$.

We can show that the power of ϵ associated with a Feynman diagram always increases with the number of loops and the number of higher-order vertices. One way to see this is to rescale the fields in the effective lagrangian so that the kinetic terms are scale invariant under the transformation $(x_0,x_{||},x_\perp)\to(\epsilon^{-1}x_0,\epsilon^{-2/3}x_{||},\epsilon^{-1/3}x_\perp)$. The scaling behavior of the fields is $\psi\to\epsilon^{5/6}\psi$ and $A_i\to\epsilon^{5/6}A_i$. We find that the scaling dimension of all interaction terms is positive. The quark gluon vertex scales as $\epsilon^{1/6}$, the HDL three gluon vertex scales as $\epsilon^{1/2}$, and the four gluon vertex scales as ϵ. Since higher order diagrams involve at least one pair of quark gluon vertices the expansion involves positive powers of $\epsilon^{1/3}$ and the low energy regime is completely perturbative.

2. Migdal Theorem

As a simple application consider the one-loop correction to the quark-gluon vertex for a gluon in the non-Fermi liquid regime, see Fig. 1. According to the scaling rules discussed

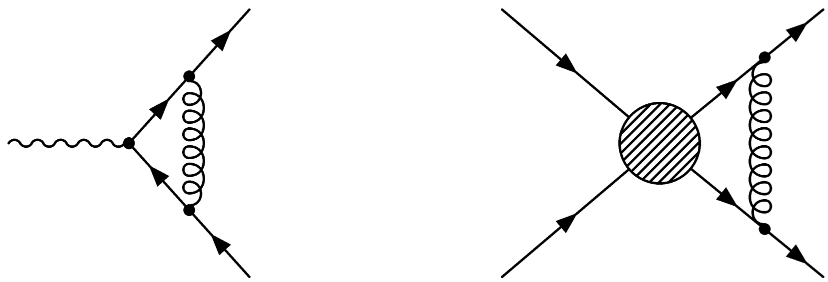

Figure 2. One-loop correction to the vertex of an external current and the BCS interaction. Both diagrams are kinematically enhanced and scale as $\log(\omega)$ and $\log^2(\omega)$, respectively.

in the last section this correction is dominated by the Abelian diagram, and scales as

$$\Gamma^a_\mu = g v_\mu (\lambda^a/2) \left(1 + O(\epsilon^{1/3})\right) . \tag{6}$$

This is a QCD version of Migdal's theorem, which states that the renormalization of the electron-phonon vertex is suppressed by the ratio $\sqrt{m/M}$, where m is the mass of the electron and M is the mass of the ions [4]. This factor is analogous to the small paramater in the QCD case, because it controls the ratio of the phonon velocity to the Fermi velocity of the electrons, and therefore also the ratio of the typical electron momentum to the typical phonon momentum.

We note that it is essential the the gluons are spacelike. Consider the vertex of an external gauge field with coupling e. The one-loop correction in the regime of small time-like momenta is [5]

$$\Gamma_\mu(p_1, p_2) = \frac{eg^2}{9\pi^2} v_\mu \log\left(\frac{\Lambda}{\omega}\right) , \tag{7}$$

where $p_{1,2}$ are the momenta of the two quarks, $\omega = (p_1^0 + p_2^0)$ and the scale Λ was determined in [2]. In this regime higher order corrections are large and have to be resummed. This is exactly what also happens in the BCS and zero-sound channels: Propagators are kinematically enhanced, and all two-body ladders have to resummed. Non-planar diagrams, or Green functions with more legs are perturbative and follow the scaling rules discussed in the previous section [2].

3. Kohn-Luttinger Theorem

One of the applications of the non-Fermi liquid effective theory is the calculation of higher order correction to the superfluid gap. The theory can also be used to study the possibility of exotic kinds of superfluidity. Kohn and Luttinger showed that for ordinary Fermi liquids superfluidity takes place even if the basic particle-particle interaction is repulsive in all channels [6]. We recently studied whether this mechanism also operates in gauge theories and leads to superfluidity in a cold electron gas [7]. The gap equation is

$$\Delta_0 = -\frac{e^2}{8\pi^2}\int \frac{d\omega\,\Delta(\omega)}{\sqrt{\omega^2+\Delta(\omega)^2}}\; f_l(\omega) \tag{8}$$

where $f_l(\omega)$ is the scattering amplitude of a $(p_F, -p_F)$ electron pair with energy ω and angular momentum l. We find that for any fixed ω the amplitude $f_l(\omega)$ becomes attractive at large l. This is the Kohn-Luttinger effect. However, for fixed l the gauge theory amplitude is always repulsive at very small ω and superconductivity does not take place.

4. Luttinger Theorem

Luttinger showed [8] that in a Fermi liquid the relationship between the density and the Fermi momentum is given by the free Fermi gas result

$$\frac{N}{V} = g_d\int\frac{d^3p}{(2\pi)^3}\,\Theta(p_F-p) \tag{9}$$

even if the system is interacting. Here, g_d is the degeneracy and p_F can be defined as the momentum at which the inverse propagator $S^{-1}(\omega\!=\!0,p)$ changes sign. The derivation of this result is discussed in standard textbooks on many body theory [9]. The main step is

$$\frac{N}{V} = ig_d\int\frac{d^3p}{(2\pi)^3}\int_{-\infty}^{0}\frac{d\omega}{2\pi}\frac{\partial}{\partial\omega}\log\left(\frac{S(\omega,p)}{S_{ret}(\omega,p)}\right) = -\frac{g_d}{\pi}\int\frac{d^3p}{(2\pi)^3}\left[\varphi(0,p)-\varphi(-\infty,p)\right] \tag{10}$$

where S_{ret} is the retarded propagator and φ is its complex phase. Luttinger's theorem follows if the phase goes from 0 to π as ω goes from 0 to $-\infty$ for the occupied states, and remains zero for unoccupied states. In a gauge theory the self energy has a cut at $\omega = 0$, $\Sigma(\omega) = c_1\omega\log(\omega) + c_2\omega^{4/3} + O(\omega^{5/3})$. As a result there is no Fermi surface in the ordinary sense as the Fermi velocity and the quasi-particle renormalization factor go to zero as $\omega \to 0$. However, Luttinger's theorem remains valid because the fermion propagator changes sign at $p = p_F$ and it acquires a phase $\varphi = \pi$ for $p < p_F$ [10]. Luttinger's theorem is not directly applicable in the superfluid phase.

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